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EDITORIAL

It is my proud privilege to welcome you all to the Researchfora International Conference at Hamburg, Germany in association with The IIER. I am happy to see the papers from all part of the world and some of the best paper published in this proceedings. This proceeding brings out the various Research papers from diverse areas of Science, Engineering, Technology and Management. This platform is intended to provide a platform for researchers, educators and professionals to present their discoveries and innovative practice and to explore future trends and applications in the field Science and Engineering. However, this conference will also provide a forum for dissemination of knowledge on both theoretical and applied research on the above said area with an ultimate aim to bridge the gap between these coherent disciplines of knowledge. Thus the forum accelerates the trend of development of technology for next generation. Our goal is to make the Conference proceedings useful and interesting to audiences involved in research in these areas, as well as to those involved in design, implementation and operation, to achieve the goal.

I once again give thanks to the Institute of Research and Journals, Researchfora, TheIIER for organizing this event in Hamburg, Germany. I am sure the contributions by the authors shall add value to the research community. I also thank all the International Advisory members and Reviewers for making this event a Successful one.

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ZERO - ONE INFLATED POISSON – SUSHILA DISTRIBUTION AND ITS APPLICATION

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Abstract - In this paper, we present a zero – one inflated Poisson – Sushila (ZOIPS) distribution which is discrete distribution. Several statistical properties of ZOIPS distribution are explored, such as the probability mass function (pmf), moment about the origin, mean and variance. Maximum likelihood estimation of the parameters is investigated. An application of the distribution to a real data set is presented finally and compared with the fit attained by some other well-known distributions for count data.

Index Terms - Poisson - Sushila distribution; Zero-one inflated model; Discrete distribution.

I. INTRODUCTION

The Poisson distribution is a well -know distribution to fit count data in practice. However theoretical prediction may not match empirical observations for moment of higher order due to the only one parameter, which does not allow the variance to be adjusted independently of mean [1]. For this problem, the mixed distribution has become increasingly popular as a more flexible alternative to the Poisson distribution. Especially it is doubtful whether the strict requirements, particularly independence, for the Poisson distribution will be satisfied [2]. Mixed distributions define one of the most important ways to obtain new probability distributions in applied probability and operational research [3]. One mixed distribution has been proposed in application to count data, especially mixed Poisson. Examples of mixed Poisson are Poisson-uniform [4], Poisson - lognormal [5], Poisson-inverse gamma [6], Poisson-Pareto [6] , Poisson-Lowmax [7], Poisson- gamma[1], etc. These have been proposed in fitting count data under the overdispersion.

Sushila distribution was introduced by Shanker, R et al. [8] which is a mixed distribution between exponential distribution and gamma distribution. Sushila distribution is a two-parameter continuous distribution. The properties of Sushila distribution such as moments, failure rate function, mean residual life function, stochastic orderings, estimation of parameters by the method of maximum likelihood and the method of moments. The Poison - Sushila distribution was introduced by Saratoon [9] which is a two parameter discrete distribution. Various properties such as moments, mean, variance, skewness and kurtosis and estimating parameter by using maximum likelihood estimation have been studied and shown that the Poisson -Sushila distribution is more flexible than Poisson distribution in real data.

An extension to the zero-inflated power series distributions or zero - one inflated models was introduced by Alshkaki [10]. Poisson distribution is one of power series distribution. An extension to the zero-inflated power series distributions is discrete distributions with number of frequencies with zero and one are inflated. Alshkaki [10] studied its structure properties; its mean, variance, probability generating function.

Zero - one inflated Poisson distribution was introduced by Alshkaki [11]. The zero - one inflated Poisson distribution is an extension to the case of zero inflated Poisson distribution. Its structural properties were studied such as mean, variance and estimation of its parameters using the methods of moments and maximum likelihood estimators.

In this study, we introduce a new discrete distribution, which is the zero – one inflated Poisson - Sushila (ZOIPS) distribution. Additionally, we present some properties of the ZOIPS distribution are the moment about origin, mean and variance. The parameters of ZOIPS distribution are estimated by maximum likelihood method, and we also present fitting distribution between the zero inflated Poisson, zero – one inflated Poisson and ZOIPS distributions based on real data set.

II. ZERO-ONE INFLATED POISSON - SUSHILA DISTRIBUTION

This section, we present zero – one inflated Poisson - Sushila (ZOIPS) distribution.

Definition 1. Let $X \square ZOIPS(\theta, \alpha, p_0, p_1)$ be a random variable of ZOIPS distribution with parameter $\theta > 0, \alpha > 0, 0 < p_0 < 1$ and $0 < p_1 < 1$.

Theorem 1. Let $X \square ZOIPS(\theta, \alpha, p_0, p_1)$ The probability mass function (pmf) of X is given by

$$f(x) = \begin{cases} p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + 1)}{(\theta + 1)(\theta + \alpha)^2} & , x = 0 \\ p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + 1)(\theta + \alpha)^3} & , x = 1 \\ (1 - p_0 - p_1) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} & , x = 2, 3, \dots \end{cases}$$

where $\theta > 0, \alpha > 0, 0 < p_0 < 1$ and $0 < p_1 < 1$.

Proof. If X is a random variable of Poisson - Sushila , then the pmf of X can be obtained by

$$g(x) = \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}}$$

$x = 0, 1, 2, \dots, \theta > 0, \alpha > 0$.

The zero – one inflated model is an extra proportion added to proportion of zero, then the pmf of zero-one inflated model is defined by

$$f(x) = \begin{cases} p_0 + (1 - p_0 - p_1)g(x=0) & , x = 0 \\ p_1 + (1 - p_0 - p_1)g(x=1) & , x = 1 \\ (1 - p_0 - p_1)g(x) & , x = 2, 3, \dots \end{cases}$$

where $0 < p_0 < 1$ and $0 < p_1 < 1$.

Then, the pmf of the ZOIPS is obtained by substituting the probability density function of Poisson - Sushila random variable into zero – one inflated model . Finally, it can be written as

$$f(x) = \begin{cases} p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + 1)}{(\theta + 1)(\theta + \alpha)^2} & , x = 0 \\ p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + 1)(\theta + \alpha)^3} & , x = 1 \\ (1 - p_0 - p_1) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} & , x = 2, 3, \dots \end{cases}$$

Next, We display some of probability mass function of ZOIPS in Figure 1-2.

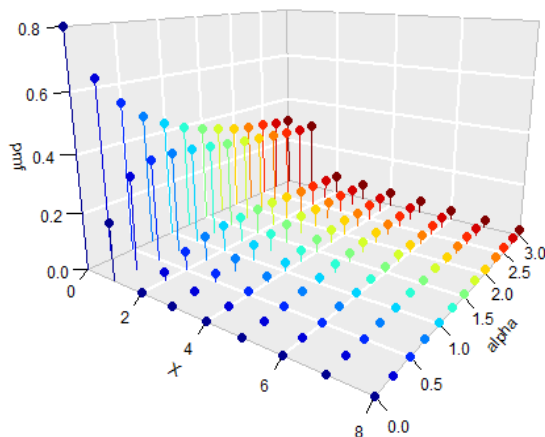


Figure 1. The pmf of a ZOIPS random variable of some values of parameters: $\theta = 1, p_0 = 0.2, p_1 = 0.2$.

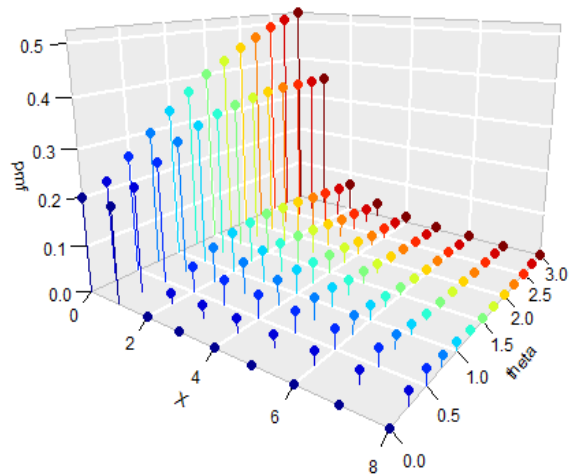


Figure 2. The pmf of a ZOIPS Random Variable of Some Values of Parameters: $\alpha = 2, p_0 = 0.2, p_1 = 0.2$.

Next, we will represent the Poisson-Sushila, zero inflated Poisson-Sushila and One inflated Poisson - Sushila distributions as a special case of the ZOIPS distribution in Corollary 1-3.

Corollary 1. If $p_0 = 0$ and $p_1 = 0$ then the ZOIPS distribution reduces to the Poisson-Sushila distribution with pmf given by

$$f(x) = \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} , x = 0, 1, 2, \dots, \theta > 0, \alpha > 0.$$

Corollary 2. If $p_0 = 0$ then the ZOIPS distribution reduces to the one inflated Poisson-Sushila distribution with pmf given by

$$f(x) = \begin{cases} p_1 + (1 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + 1)(\theta + \alpha)^3} & , x = 1 \\ (1 - p_1) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} & , x = 0, 2, 3, \dots \end{cases}$$

where $\theta > 0, \alpha > 0$ and $0 < p_1 < 1$.

Corollary 3. If $p_1 = 0$ then the ZOIPS distribution reduces to the zero inflated Poisson-Sushila distribution with pmf given by

$$f(x) = \begin{cases} p_0 + (1 - p_0) \frac{\theta^2 (\theta + \alpha + 1)}{(\theta + 1)(\theta + \alpha)^2} & , x = 0 \\ (1 - p_0) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} & , x = 1, 2, 3, \dots \end{cases}$$

where $\theta > 0, \alpha > 0$ and $0 < p_0 < 1$.

III. SOME PROPERTIES OF ZERO-ONE INFLATED POISSON - SUSHILA DISTRIBUTION

In this part, we introduce some properties of the ZOIPS distribution. We begin with moment about the origin of this distribution in Theorem 2.

Theorem 2. If $X \square ZOIPS(\theta, \alpha, p_0, p_1)$, then the r th moment about origin of X is given by

$$E(X^r) = p_1 + (1 - p_0 - p_1) \frac{r!(\theta + r + 1)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)^r$$

where $r = 1, 2, \dots, \theta > 0, \alpha > 0, 0 < p_0 < 1$ and $0 < p_1 < 1$.

Proof. From the r th moment about origin of ZOIPS can be obtained by

$$\begin{aligned} E(X^r) &= \sum_{x=0}^{\infty} x^r f(x) = \sum_{x=1}^{\infty} x^r f(x) \\ E(X^r) &= p_1 + \sum_{x=1}^{\infty} x^r (1 - p_0 - p_1) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} \\ &= p_1 + (1 - p_0 - p_1) \sum_{x=1}^{\infty} \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} \end{aligned}$$

We have that $\sum_{x=1}^{\infty} \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}}$ is the r th moment about origin of Poisson - Sushila distribution.

Then,

$$E(X^r) = p_1 + (1 - p_0 - p_1) \frac{r!(\theta + r + 1)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)^r.$$

From the r th moment about origin of ZOIPS distribution, it is straightforward to deduce the first - fourth moment about origin, mean, and variance respectively are

1) first - fourth moment about origin

$$E(X) = p_1 + (1 - p_0 - p_1) \frac{(\theta + 2)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)$$

$$E(X^2) = p_1 + (1 - p_0 - p_1) \frac{(2\theta + 6)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)^2$$

$$E(X^3) = p_1 + (1 - p_0 - p_1) \frac{(6\theta + 24)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)^3$$

$$E(X^4) = p_1 + (1 - p_0 - p_1) \frac{(10\theta + 50)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)^4$$

2) Mean moment

$$E(X) = p_1 + (1 - p_0 - p_1) \frac{(\theta + 2)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right)$$

Variance

$$3) \text{Var}(X) = p_1 + (1 - p_0 - p_1) \frac{(\theta + 2)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right) \left[p_1 + (1 - p_0 - p_1) \frac{(\theta + 2)}{(\theta + 1)} \left(\frac{\alpha}{\theta}\right) \right]^2$$

4) Mean and variance of probability mass function of ZOIPS distribution is shown in Figure 3.

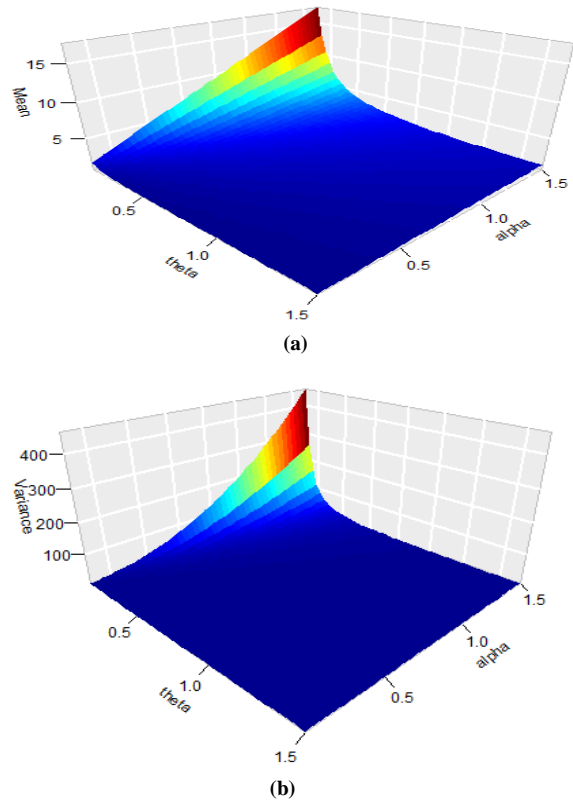


Figure 3. Mean (a) and Variance (b) of a ZOIPS Random Variable of Some Values of Parameters.

IV. PARAMETER ESTIMATION

In this section, the estimation of parameters for the ZOIPS via the maximum likelihood procedure is provided. The likelihood function of the ZOIPS $(\theta, \alpha, p_0, p_1)$ is given by

$$\begin{aligned} L(\theta, \alpha, p_0, p_1) &= \prod_{i=1}^n \left(p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + 1)}{(\theta + 1)(\theta + \alpha)^2} \right)^{a_i} \\ &\quad \times \left(p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + 1)(\theta + \alpha)^3} \right)^{b_i} \\ &\quad \times \left((1 - p_0 - p_1) \frac{\theta^2 \alpha^x (\theta + \alpha + x + 1)}{(\theta + 1)(\theta + \alpha)^{x+2}} \right)^{1 - a_i - b_i} \end{aligned}$$

where $a_i = \begin{cases} 1, & x_i = 0 \\ 0, & x_i \neq 0 \end{cases}$ and $b_i = \begin{cases} 1, & x_i = 1 \\ 0, & x_i \neq 1 \end{cases}$

with the corresponding log-likelihood function:

$$\begin{aligned} \log L(\theta, \alpha, p_0, p_1) &= \left(\sum_{i=1}^n a_i \right) \log \left(p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + 1)}{(\theta + 1)(\theta + \alpha)^2} \right) \\ &\quad + \left(\sum_{i=1}^n b_i \right) \log \left(p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + 1)(\theta + \alpha)^3} \right) \\ &\quad + \sum_{i=1}^n (1 - a_i - b_i) \log(1 - p_0 - p_1) \\ &\quad + \sum_{i=1}^n (1 - a_i - b_i) 2 \log \theta - \sum_{i=1}^n (1 - a_i - b_i) \log(\theta + 1) \\ &\quad + \sum_{i=1}^n (1 - a_i - b_i) (x_i + 2) \log(\theta + \alpha) + \sum_{i=1}^n (1 - a_i - b_i) x_i \log \alpha \\ &\quad + \sum_{i=1}^n (1 - a_i - b_i) \log(\theta + \alpha + x_i + 1). \end{aligned}$$

The optimal values of the parameters can be obtained by the first partial derivatives of log-likelihood function with respect to θ, α, p_0 and p_1 . Then, it gives rise to following equations:

$$\frac{\partial \log L}{\partial \theta} = \left(\frac{n_0 \left[\Lambda_0 (3\theta^2 + 2\theta(\alpha + I)) - (\theta^3 + \alpha\theta^2 + \theta) \frac{\partial \Lambda_0}{\partial \theta} \right]}{p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \Lambda_0^2} \right) + \left(\frac{n_1 \left[\Lambda_1 \alpha (3\theta^2 + 2\theta(\alpha + I)) - \alpha (\theta^3 + \alpha\theta^2 + \theta) \frac{\partial \Lambda_1}{\partial \theta} \right]}{p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \Lambda_1^2} \right) + \frac{2(n - n_0 - n_1)}{\theta} - \frac{n - n_0 - n_1}{\theta + I} + \sum_{i=1}^n \frac{(1 - a_i - b_i)(x_i + 2)}{\theta + \alpha} + \sum_{i=1}^n \frac{1 - a_i - b_i}{\theta + \alpha + x_i + I}$$

here $n_0 = \sum_{i=1}^n a_i$, $n_1 = \sum_{i=1}^n b_i$, $\Lambda_0 = (\theta + I)(\theta + \alpha)^2$ and

$$\Lambda_1 = (\theta + I)(\theta + \alpha)^3$$

$$\frac{\partial \log L}{\partial \alpha} = \left(\frac{n_0 \left[\Lambda_0 \theta^2 - (\theta^3 + \alpha\theta^2 + \theta) \frac{\partial \Lambda_0}{\partial \alpha} \right]}{p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \Lambda_0^2} \right) + \left(\frac{n_1 \left[\Lambda_1 (\theta^3 + 2\theta^2 \alpha + 2\theta^2) - \alpha (\theta^3 + \alpha\theta^2 + \theta) \frac{\partial \Lambda_1}{\partial \alpha} \right]}{p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \Lambda_1^2} \right) + \frac{\sum_{i=1}^n (1 - a_i - b_i)(x_i + 2)}{\theta + \alpha} + \frac{\sum_{i=1}^n (1 - a_i - b_i)x_i}{\alpha}$$

$$\frac{\partial \log L}{\partial p_0} = \left(\frac{n_0 \left(1 - \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \right)}{p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \Lambda_0^2} \right) - \left(\frac{n_1 \left(\frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \right)}{p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \Lambda_1^2} \right) - \left(\frac{n - n_0 - n_1}{1 - p_0 - p_1} \right)$$

$$\frac{\partial \log L}{\partial p_1} = \left(\frac{-n_0 \left(\frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \right)}{p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \Lambda_0^2} \right) + \left(\frac{n_1 \left(1 - \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \right)}{p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \Lambda_1^2} \right) - \left(\frac{n - n_0 - n_1}{1 - p_0 - p_1} \right)$$

$$\frac{\partial \log L}{\partial p_1} = \left(\frac{-n_0 \left(\frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \right)}{p_0 + (1 - p_0 - p_1) \frac{\theta^2 (\theta + \alpha + I)}{(\theta + I)(\theta + \alpha)^2} \Lambda_0^2} \right) + \left(\frac{n_1 \left(1 - \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \right)}{p_1 + (1 - p_0 - p_1) \frac{\theta^2 \alpha (\theta + \alpha + 2)}{(\theta + I)(\theta + \alpha)^3} \Lambda_1^2} \right) - \left(\frac{n - n_0 - n_1}{1 - p_0 - p_1} \right)$$

These can be solved numerically by using Newton-Raphson method. In this paper, we obtain

the MLE estimates of $\hat{\theta}, \hat{\alpha}, \hat{p}_0$ and \hat{p}_1 . by using function nlm in stats package of R program.

V. APPLICATION

The ZOIPS distribution is applied on count data. We use a real data set which is the number of stillbirths in 402 litters of New Zealand white rabbits, originally used by Morgan et al [12]. The real data set is fitted by the zero inflated Poisson (ZIP), zero - one inflated Poisson (ZOIP) and ZOIPS distributions. The maximum likelihood method provides parameter estimations. The performances of the model fittings by using chi-squared statistic and Akaike information criterion (AIC) in Table 1. The results show that the ZOIPS distribution provides a better fit than the ZIP and ZOIP distributions.

CONCLUSION

In this study, we introduce the ZOIPS distribution which is an extra proportion of zero of the random variable Poisson - Sushila. In particular, the closed form, first - fourth moment about origin, mean, and variance of the ZOIPS distribution are derived. In addition, the parameter estimations are shown via the maximum likelihood method. For application to real data set, it shows that this distribution can provide a better fit than other parent distributions. We hope that the ZOIPS distribution may be an alternative model of count data analysis with many zero and one.

Number of Stillbirths	Observed	Expected by ZIP	Expected by ZOIP	Expected by ZOIPS
0	314	314	314	314
1	48	36	48	48
2	20	28	16	18
3	7	15	11	10
4	5	} 9	} 6	} 6
5	2			
6+	6			
Estimated parameters		$\hat{\lambda} = 1.58$	$\hat{\lambda} = 2.10$	$\hat{\theta} = 1.57$
		$\hat{p}_0 = 0.72$	$\hat{p}_0 = 0.77$	$\hat{\alpha} = 1.44$
			$\hat{p}_1 = 0.08$	$\hat{p}_0 = 0.65$
				$\hat{p}_1 = 0.04$
Chi-squared		35.72	3.95	1.66
Degree of freedom		3	2	1
p-value		<0.0001	0.0467	0.1973
AIC		686.58	672.58	671.76

Table 1. Estimated parameters of ZIP, ZOIP and ZOIPS for number of stillbirths of New Zealand white rabbits data.

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