FLIPPED CLASSROOM AND THE GEOMETER'S SKETCHPAD: STUDENTS' INVESTIGATION-A SQUARE PEG IN A ROUND HOLE

Asst.Prof. Dr. Krongthong Khairiree

International College, Suan Sunandha Rajabhat University,
Bangkok, Thailand.
krongthong.kh@ssru.ac.th or drkrongthong@gmail.com

ABSTRACT

The purpose of this study was to explore the teaching approaches using a flipped classroom incorporated with the dynamic software program the Geometer's Sketchpad. In the 2018, action research was conducted in mathematics class of International College, Suan Sunandha Rajabhat University, Thailand. In the flipped classroom, the researcher created online lessons of her lectures and the students viewed them prior to attending class. Students worked on problem-solving activities in their classrooms. The students used the Geometer's Sketchpad to investigate on the problem "Which fits best, a square peg in a round hole or a round peg in a square hole? The research findings indicated that the flipped model of instruction was new teaching strategy that moved the lecture on mathematics outside classroom via technology and brought assignments/exercises of mathematics concepts inside the classroom via learning activities. The research findings shown the students engagement in the flipped classroom were higher than the using traditional classroom. Based on the students' interviews they revealed that using flipped classroom incorporated with Geometer's Sketchpad method they were able to make learning mathematics fun and challenging.

Keywords: Flipped classroom, Geometer's Sketchpad, Mathematical problem solving, Students' engagement

INTRODUCTION

International College, Suan Sunandha Rajabhat University (SSRUIC) has the purpose that the teaching and learning process at SSRUIC has to be improved in order to raise the quality of the Graduate. With the intention of encouraging the students' involvement in their learning during classes and after classes, the courses offer at SSRUIC have to be student centered learning, constructivist approach, problem-based learning and using Information and communication Technology. SSRUIC lecturers have to find the meaningful problem-based learning to enhance students to do to learn inside classroom and outside classroom. As such, this action research was conducted in order to explore the teaching approaches using a flipped classroom incorporated with the Geometer's Sketchpad in mathematics subject.

Normally in a traditional mathematics class, a lecturer conducted lesson in the following sequence: a lecturer would begin the lesson by briefly presenting the mathematics concepts, explaining the formula or procedures following by working out one or two examples as illustration for students in the class. The lecturer would then assign some time in class for students to work out similar problems and additional exercises would be given as homework to reinforce the learning achieved. If the mathematics course was to be student centered learning, constructivist approach, problem-based learning and using ICT, then the tradition teaching and learning had to be changed. This provided a challenge to the lecturer to change her teaching method. Mathematics is one of the subjects that most students learn without understanding. The reason might be that with mathematics, there is an overwhelming emphasis on the manipulation of symbols and formulas, and because of that it has been too abstract for many students. Teachers expect their students to spend large amounts of time attempting to paper-and-pencil algorithms associated with problem solving. Students are required to memorize formula and algorithmic computation, but they do not really understand and quickly forget them (Skemp, 1978). This is because learning mathematic is more than computation, more than memorizing rules and facts. It is investigation, exploring, experimenting, posing problems and solving problems. Students have to learn and understand the language, including the symbolisms, and grasp basic concepts.

Flipped Classroom

In 2012, Jonathan Bergmann and Aaron Sams (2012) from U.S.A. created the new method of teaching, they recorded their teaching lesson, PowerPoint slide showed including voice and any annotations, converted the recording into a video file and then uploaded online. They believed that recorded lectures might be a way to keep their students who missed class could access them. Based on Bergmann and Sams' research findings (2012), the absent students loved the recorded lectures. With the video recorded lectures the students were able to watch the video many times at their own pace until they mastered the content. In a flipped classroom, students watch a video lesson at home and come to class for hands-on activities and real life applications. Bergmann and Sams revealed that flipped classroom model was more efficient than gave lecture and assigned homework. They indicated that the flipped classroom was a better model than the traditional approach. However, they explained the important facts that there is no single way to flip your classroom. Flipping the classroom is more about a mindset: redirecting attention away from the teacher and putting attention on the learner and the learning. The flipped classroom is a pedagogical model in which the typical lecture and homework elements of a course are reversed. Student watched a short video lectures at home before the class session, while the in-class time the students do the exercises, discussions or projects.

The Geometer's Sketchpad (GSP)

The Geometer's Sketchpad is one of the dynamic mathematics software that provides opportunities for students to investigate and discover mathematics concepts in particular geometric patterns. GSP empowers students to use their abilities to create graphical representation, to enable them in developing their mathematical thinking skills, concepts, and understanding. Khairiree (2011) described that while using GSP students learned by exploring, investigating and discovering. GSP enhance students' ability in helping them visualize abstract mathematical relationships and various problem structures through pictorial representations.

Mathematical Problem Solving

Students were assigned to investigate on the mathematical problem solving. "Which fits better, a square peg in a round hole or a round peg in a square hole? A square peg in a round hole is an expression that has been in use since about 1800 for a misfit, nonconformist or eccentric, someone who does not fit well into society. The situation where persons feel that they are in the wrong place, or doing an activity for which they are unsuitable or do not fit in with the people around them. The van Hiele theory was employed in this study. The van Hiele theory of geometric thought had great impact on the geometric thinking (van Hiele & van Hiele-Geldof, 1958). van Hiele theory consists of five-level hierarchy of ways of understanding of geometry. They are visualization, analysis, informal deduction, deduction and rigor. Each of the five levels describes the thinking processes used in geometric contexts. van Hiele describe that the product of thought at each level are the same as the objects of thought at the next level. The students have to create their ideas at one level so that relationships among these ideas can become the focus of the next level. Moreover Meyer (2001) explains that students bridge the gap between concrete and increasingly abstract levels through their creation and use of models, drawings, diagrams, tables or symbolic notations and technology is a good tool for mathematical modeling. The students also used the Geometer's Sketchpad to investigate on this problem.

Flipped Classroom, GSP and Action Research in Thailand

In Fiscal Year 2018, the author conducted the action research in mathematics class in order to explore the classroom environment of using a flipped classroom incorporated with the Geometer's Sketchpad in solving mathematics problem. The research study was conducted in March 2018. The subjects were 32 first year students of SSRUIC, Bangkok, Thailand. They are at 19-year-old students. The flipped classroom teaching approach was employed in this study. Students had to study and work on mathematics problems before attended mathematics class.

Research Questions

1. What are the components of teaching and learning instruction using flipped classroom and the Geometer's Sketchpad method?
2. How Geometer's Sketchpad enhance students' investigation in solving mathematics problems in Flipped classroom?

Research Question 1: What are the components of teaching and learning instruction using flipped classroom method?

RESEARCH FINDINGS

Based on the research findings, in order to implement flipped classroom and the Geometer's Sketchpad method effectively the components of teaching and learning instruction have to include two topics as follows:

(1) Mathematics Course Webpage and Moodle of SSRUIC for students to search information, to follow up lessons and work assignments. The components of the webpage must included information as follows:

- Students Online Users name;
- Thailand Qualification Framework 3 (TQF 3), Course outline, lesson plan, and learning objectives;
- Handout and lecture notes;
- Weekly video tape recorded of the lesson conducted during classes;
- You Tube of the relevance topics;
- Assignments and problem-based learning questions
- Quiz/Assessment/Evaluation and Web Link.

(2) The students have to learn how to use the Geometer's Sketchpad software program effectively. The Geometer's Sketchpad online activities must embedded in the Webpage throughout the course duration.

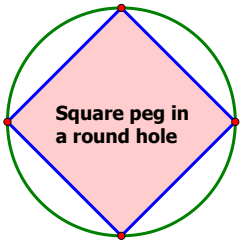
Research Question 2: How Geometer's Sketchpad enhance students' investigation in solving mathematics problems in Flipped classroom?

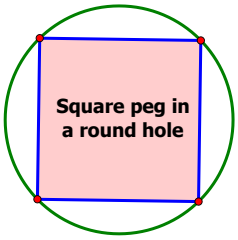
In a flipped classroom, the students learned the topics of area formulas, Pythagoras theorem, and using algebra in solving mathematics problem. The researcher posted the problem on the webpage and moodle of SSRUIC course: *Which fits best, a square peg in a round hole or a round peg in a square hole?* The students were assigned to draw a diagram to represent a physical for a general case, apply Pythagoras Theorem to a general situation. Then devise and use problem solving strategies to investigate situations mathematically. The students have to study and find the way to solve this problem before attend the class. This problem requires careful thinking and investigating. The students worked together in a small group, they discussed what "best fit" means and decided what quantities will be needed in order to do the comparison. The drawing strategy in solving this word problems was employed as a strategy in mathematics classes.

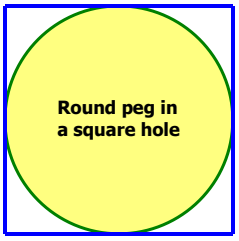
The following examples show students activities in using the Geometer's Sketchpad to investigate on the problem: *Which fits best, a square peg in a round hole or a round peg in a square hole?*

1) The students decided on and sketched the best diagrams for a well-fitting *round peg* in a square hole and *square peg* in a round hole. The students discussed on the criterion for best fit was the critical mathematical modelling. What really is the right quantity to compare? One criterion could be the best fit is touching more of the sides of the hole or a tighter fit. The students look for the one that fill up the hole the most completely.
   This can be measured by the proportion of the area of the hole that is blocked by the peg. The students also showed their solutions by expressing the area of the inner shape as percentage or

fraction of the whole area. The diagrams below showed the inside circle or square touching the outside shape at four points. The orientation of the square in the round hole could vary.



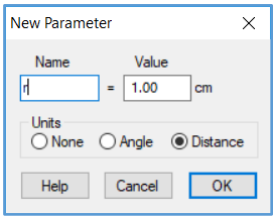




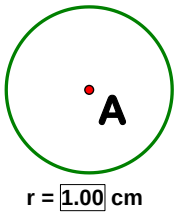
2) The students worked together in a small group, each group choose a value for the radius of the circle or the side length of the square in each diagram to begin. They applied Pythagoras Theorem to general situation, they devised and used problem solving strategies to explore situations mathematically. The students worked from a specific radius measurement. They used GSP to sketch the circle and the square as follows.

(1) Students constructed "**Square peg** in a round hole" by using GSP step by step as following:

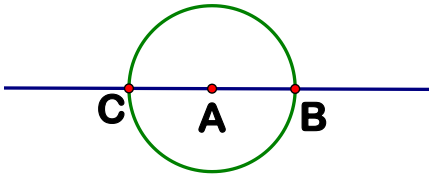
- Open the Geometer's Sketchpad Version 5, in the File Menu, choose New Sketch.
- Construct parameter *r*
  - Select **New Parameter** in the **Number menu**;
  - Type ***r*** and value of radius = 1.00 in the pop up menu as shown on the right; and
  - Select **Distance Units**: ***cm*** as shown on the right.



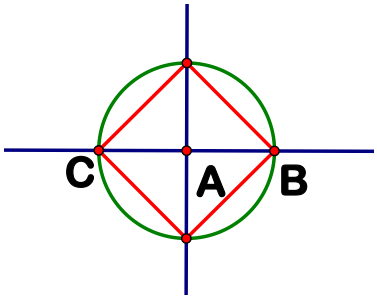
- Construct circle by point and radius:
  - Construct point ***A*;**
  - Select point ***A*** and the parameter ***r***, then in the construct menu choose **Circle By Center + Radius.**



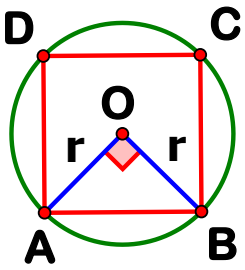
- Construct point B on a circumference of circle A;
- Construct a line through point A and point B:
  - Select point A and point B, then in the construct menu choose **Line**.
- Line AB intersects a circumference at point C.



- Select point A, in the Transform menu choose **Mark Center**.
- Select line AB, point B and point C, in the Transform menu choose **Rotate**;
- Choose By **fixed angle** and enter **90º** and then click **OK**;
- Construct four segments to sketch a square as show on your right.



- Select all lines, in the Display menu choose Hide lines;
- Change Name of four points of a square to ABCD, name the center of a circle as point O;
- Construct radius AO and BO; and
  ∠ AOB = 90 °
- Drag point **B** around the circle and
  the orientation of the square in the circle could vary.



If the radius of the circle is ***r***, then by Pythagoras Theorem

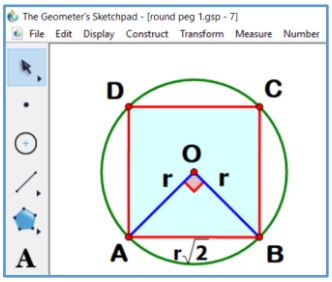
$AB^2 = AO^2 + BO^2$

$AB^2 = r^2 + r^2$

$AB^2 = 2r^2$

the side of the square AB $= r\sqrt{2}$

Hence the area of square ABCD $= (r\sqrt{2}) \times (r\sqrt{2}) = 2r^2$

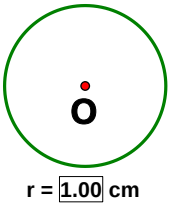


The area of the circle is $= \pi r^2$

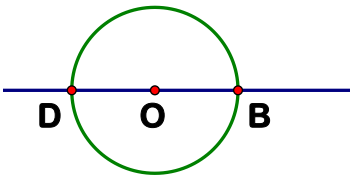
(2) Students constructed "**Round peg** in a square hole" by using GSP step by step as following:

Students used GSP to construct a circle and a square as follows:

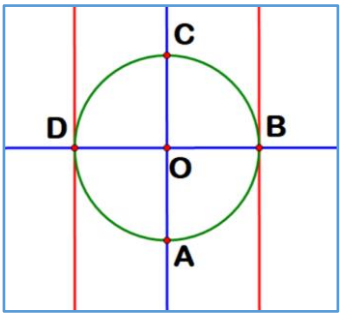
- Construct circle by point and radius:
  - Construct point **O**;
  - Select point **O** and the parameter ***r***, then
    in the construct menu choose **Circle By Center + Radius**.



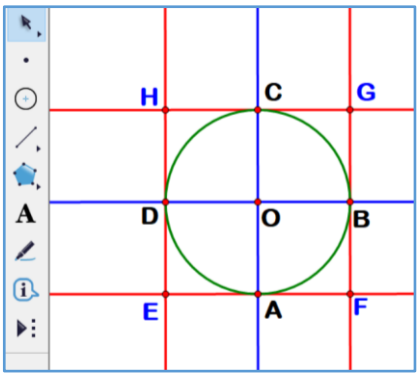
- Construct point B on a circumference of circle O;
- Construct a line through point O and point B:
  - Select point O and point B, then
    in the construct menu choose line;
- Line OB intersects a circumference at point D;



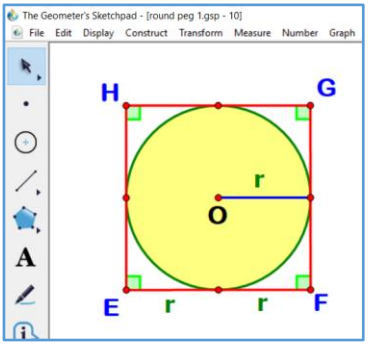
- Select point O, in the Transform menu choose
  Mark Center;
- Select line DB, point B and point D, in the Transform menu choose **Rotate**;
- Choose By **fixed angle** and enter 9**0º** and then click **OK**;
- The rotation of line DB intersects a circumference at point A and point C;
- Construct a line through point B perpendicular to line DB;
- Construct a line through point D perpendicular to line DB;



- Construct a line through point A parallel to line DB;
- Construct a line through point C parallel to line DB;



- Construct radius OB and circle interior;
- Construct four segments to make a square EFGH and hide all lines as show on your right.



The side of the square EFGH = r + r = 2r
Hence the area of square EFGH = 2r × 2r = $4r^2$

Area of the circle is = πr^2

3) The students discussed how they will judge which fits best. The possible way included express the area of the inner shape as a percentage or fraction of the area of the outer shape. The tables below shows the example of students investigated and showed their works from a specific radius measurement.

Table 1 Percentage of Comparison of the Area of the Inner Shape and the Outer Shape when r = 1

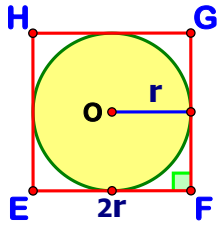
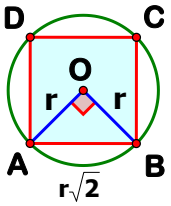
For the round peg in a square hole		For the square peg in a round hole	
	**r = 1 unit** Square Hole: Area of square = $4r^2$ $= 4 \times 1^2 = 4$ Round Peg: Area of circle = πr^2 $= \pi 1^2 = \pi$		**r = 1 unit** Round Hole: Area of circle = πr^2 $= \pi 1^2 = \pi$ Square peg: Area of square = $2r^2$ $= 2 \times 1^2 = 2$
Fraction of the square hole filled by the round peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ round\ peg}{Area\ of\ square\ hole} = \frac{\pi}{4}$ $\cong 0.79$ Approximately 79 % of the hole is filled by the round peg.		Fraction of the round hole filled by the square peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ square\ peg}{Area\ of\ round\ hole} = \frac{2}{\pi}$ $\cong 0.64$ Approximately 64 % of the hole is filled by the square peg.	

Table 2 Percentage of Comparison of the Area of the Inner Shape and the Outer Shape when r = 5

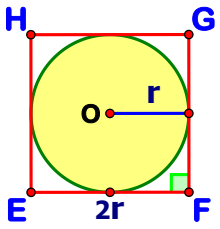
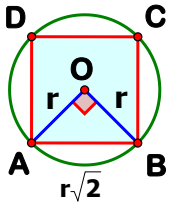
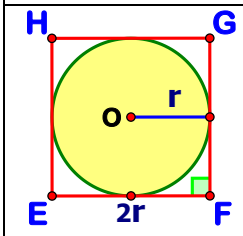
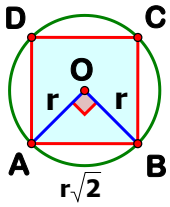
For the round peg in a square hole		For the square peg in a round hole	
	**r = 5 unit** Square Hole: Area of square = $4r^2$ $= 4 \times 5^2 = 100$ Round Peg: Area of circle $= \pi r^2$ $= \pi 5^2 = 25\pi$		**r = 5 unit** Round Hole: Area of circle $= \pi r^2$ $= \pi 5^2 = 25\pi$ Square peg: Area of square = $2r^2$ $= 2 \times 5^2 = 50$
Fraction of the square hole filled by the round peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ round\ peg}{Area\ of\ square\ hole} = \frac{25\pi}{100}$ $\cong 0.79$ Approximately 79 % of the hole is filled by the round peg.		Fraction of the round hole filled by the square peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ square\ peg}{Area\ of\ round\ hole} = \frac{50}{25\pi}$ $\cong 0.64$ Approximately 64 % of the hole is filled by the square peg.	

Table 3 Percentage of Comparison of the Area of the Inner Shape and the Outer Shape when radius = r

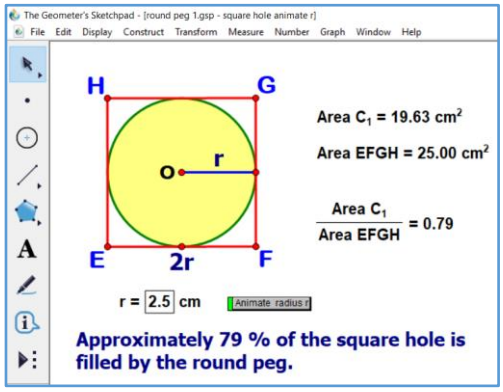
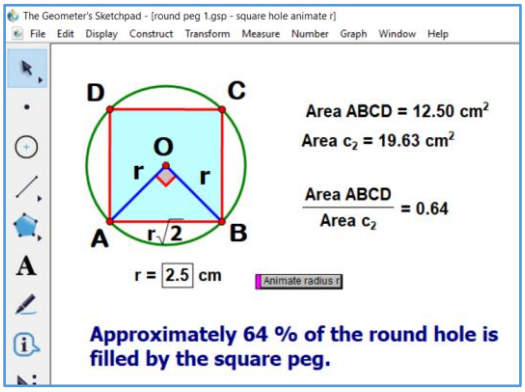
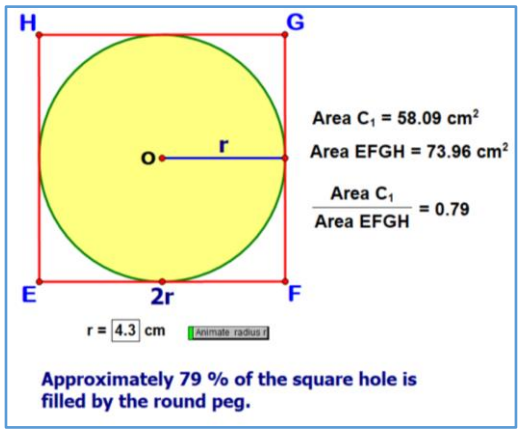
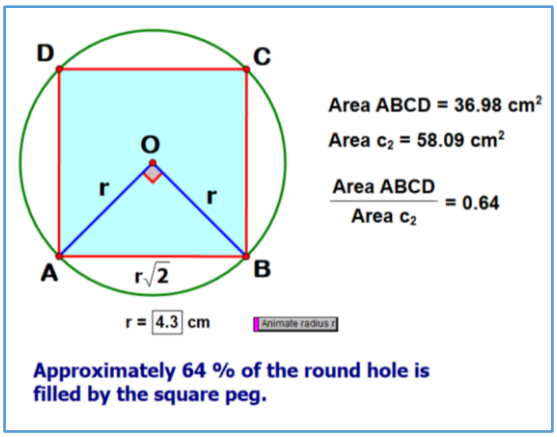
For the round peg in a square hole		For the square peg in a round hole	
	**radius = r unit** Square Hole: Area of square = $4r^2$ Round Peg: Area of circle $= \pi r^2$		**radius = r unit** Round Hole: Area of circle $= \pi r^2$ Square peg: Area of square = $2r^2$
Fraction of the square hole filled by the round peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ round\ peg}{Area\ of\ square\ hole} = \frac{\pi r^2}{4r^2}$ $= \frac{\pi}{4}$ $\cong 0.79$ Approximately 79 % of the hole is filled by the round peg.		Fraction of the round hole filled by the square peg $= \frac{Area\ of\ Inner\ Shape}{Area\ of\ Outer\ Shape} = \frac{Area\ of\ square\ peg}{Area\ of\ round\ hole} = \frac{2r^2}{\pi r^2}$ $= \frac{2}{\pi}$ $\cong 0.64$ Approximately 64 % of the hole is filled by the square peg.	

4) The students used the Geometer's Sketchpad to animate the radius of circle to investigate which fits best by varying radius ***r*** and compare the fraction of the area of the inner shape to the area of the outer shape.

Animate radius r:

- Select parameter **r**, then in the Display menu choose Animate Distance parameter ***r*** or
- Animate parameter **r** by creating an action button
  - Select parameter **r**, Choose Edit Menu, Action button, Animation and click OK.

Table 4 Percentage of Comparison of the Area of the Inner Shape and the Outer Shape by using GSP

For the round peg in a square hole	For the square peg in a round hole
	
	

Based on the students'investigation of various values of the radius ***r*** from Table 1 to Table 4, the students worked with numerical examples, they chose the size of the hole or pegs. The students observed the general result and move to algebra to demonstrate it. They found that the percentage of area covered does not depend on the size of the hole. The students explained that a round peg filled up about 79% of a square hole, where as a square peg only filled up about 64% of a round hole. The students' conclusion and answer to this problem: Which fits best, a square peg in a round hole or a round peg in a square hole? is:

"A round peg in a square hole is a better fit than a square peg in a round hole."

CONCLUSION

Based on the research findings indicated that the flipped model of instruction was new teaching strategy that moved the lecture on mathematics outside classroom via technology and brought assignments/exercises of mathematics concepts inside the classroom via learning activities. The research findings shown the students engagement in the flipped classroom were higher than the using traditional classroom. Based on the students' interviews they revealed that using flipped classroom incorporated with Geometer's Sketchpad method they were able to make learning mathematics fun and challenging.

REFERENCES

Bennett, D. (2002). *Exploring geometry with the Geometer's Sketchpad: Revised for use with Version 4.* CA: Key Curriculum Press.

Bergmann, J. & Sams, A. (2012). *Flip Your Classroom: Reach Every Student in Every Class Every Day.* ISTE: International Society for Technology in Education. Eugene: U.S.A.

Blythe, T. (1998). *Teaching for understanding guide.* San Francisco, CA: Jossey-Boss Inc.

Colm, C. (2018). In What Flipping Dimension is a Square Peg in a Round Hole just as Good as a Round Peg in a Square Hole? Retrieve date May 20, 2018 from http://digg.com/video/round-peg-square-hole-numberphile

Dougiamas, M. (2001). Moodle: open-source software for producing internet-based courses. Retrieved February 18, 2011, from http://moodle.com/

Hoffer, A. (1981). Geometry is more than proof. *Mathematics Teacher*, NCTM, 74, 11-18.

Kimberling, C. (2003). *Geometry in action: A discovery approach using The Geometer's Sketchpad.* Emeryville CA: Key College Publishing.

Khairiree, K. (2011). Constructivist in Mathematics in Virtual Class with Moodle and the Geometer's Sketchpad. In *Proceeding of The Sixteenth Asian Technology Conference in Mathematics.* Wei-Chi Yang, Miroslaw Majewski, Tilak de Alwis, & Erol Karakirk (Eds). Abant Izzet Baysal University, Bolu, Turkey. Mathematics and Technology, LLC. Published, U.S.A.

Meyer, M.R. (2001). Representation in realistic mathematics education. In A.A. Cuoco and F.R. Curcio (Eds.), *The roles of representation in school mathematics. 2001 Yearbook.* pp.238-250. National Council of Teachers of Mathematics. Reston: VA

Pritchard, C. (2016). A Square Peg in a Round Hole Adventures in the Mathematics of Area. The Mathematical Association.

Sierpinska, A. (1994). *Understanding in mathematics.* London: The Falmer Press.

Singmaster, D. (1964). On Round Peg in Square Holes and Square Pegs in Round Holes (Math.Mag. Vol.37: 335-337). Retrieve date May 20, 2018 from www.jstor.org/stable/2689251

Skemp, R.R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher*, 26(3), 9-15.

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge to reform. *Journal for Research in Mathematics Education*, 27, 387–402.

Van Hiele, P.M. (1986). *Structure and insight: A Theory of mathematics education.* Orlando, FL: Academic Press.

van Hiele, P.M., & van Hiele-Geldof, D. (1958). A method of initiation into geometry. In H. Freudenthal (Ed.), *Report on methods of initiation into geometry.* Groningen: Wolters.